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J. Phys. A: Math. Gen. 20 (1987) 6627-6631. Printed in the UK

COMMENT

Distribution of first-passage times for diffusion at the percolation threshold

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Received 27 April 1987, in final form 11 June 1987

Abstract. Simulations for dispersion of diffusion at the percolation threshold of triangular and Bethe lattices show scaling behaviour. With 'topological' bias we find a maximum of the arrival time distribution at short times, a power-law decay for intermediate times and an exponential decay for long times.

If fluids flow through a porous medium, different parts of the fluid take different amounts of time to flow the same distance (dispersion). One model for dispersion is diffusion on percolating clusters [1-5], where a random walker can move only on occupied sites. This walk is called biased if one direction is taken more often than the others. This direction can be fixed in space [6], oriented away from the origin ('topological') [7], oriented along the current flow direction [8, 9], or it can be random [10]. The case of topological bias seems numerically and analytically best understood [7] and thus is chosen for the present study.

Therefore we check how long a random walker needs to travel a 'chemical distance' l, i.e. to move to a site separated by l nearest-neighbour bonds (within the percolating cluster) from the origin of the walk. $P(t)$ is the probability that the walker arrives there first after t steps. In general, a step which increases the chemical distance l from the origin is taken with a probability proportional to $1+E$, a step in the opposite direction with probability proportional to $1-E$. This bias field may correspond to the pressure gradient in a porous medium, if a fluid is injected at the origin. We simulate this dispersion problem on a computer at the critical concentration $p=p_c=\frac{1}{2}$ of a triangular and a Bethe lattice (Cayley tree). The random medium is produced by Monte Carlo methods, the diffusion process on it by exact enumeration [2].

Figure 1 shows that the histogram $P(t)$ of first-arrival times obeys a scaling law even for moderately large distances l. The RMS fluctuation $(\langle t^2\rangle-\langle t\rangle^2)^{1/2}$ is about as large as the average $\langle t\rangle$. We plot double logarithmically the ratio $\pi(t)=P(t)/P(t_{\max})$ against $t/t_{1/2}$. Here $t_{\max}$ is the time at which $P(t)$ reaches its maximum, and $t_{1/2}$ the later time after which $P(t)$ has decayed to half its maximum value. This way of plotting avoids any assumptions on how the times depend on the length l. The inserts in figure 1 show that $t_{\max}$ and $t_{1/2}$ increase roughly as $l^{2.4}$ on the triangular lattice and as $l^{2.6}$ on the Cayley tree. Theoretically we expect [2] these exponents to be about $d_w^l=2.5$ and $d_w^l=3$ for $t\to\infty$.

We see an impressive agreement between the triangular and Bethe lattices. For example, the ratio $t_{1/2}/t_{\max}$ is about 3 in the triangular lattice and only 10% larger in

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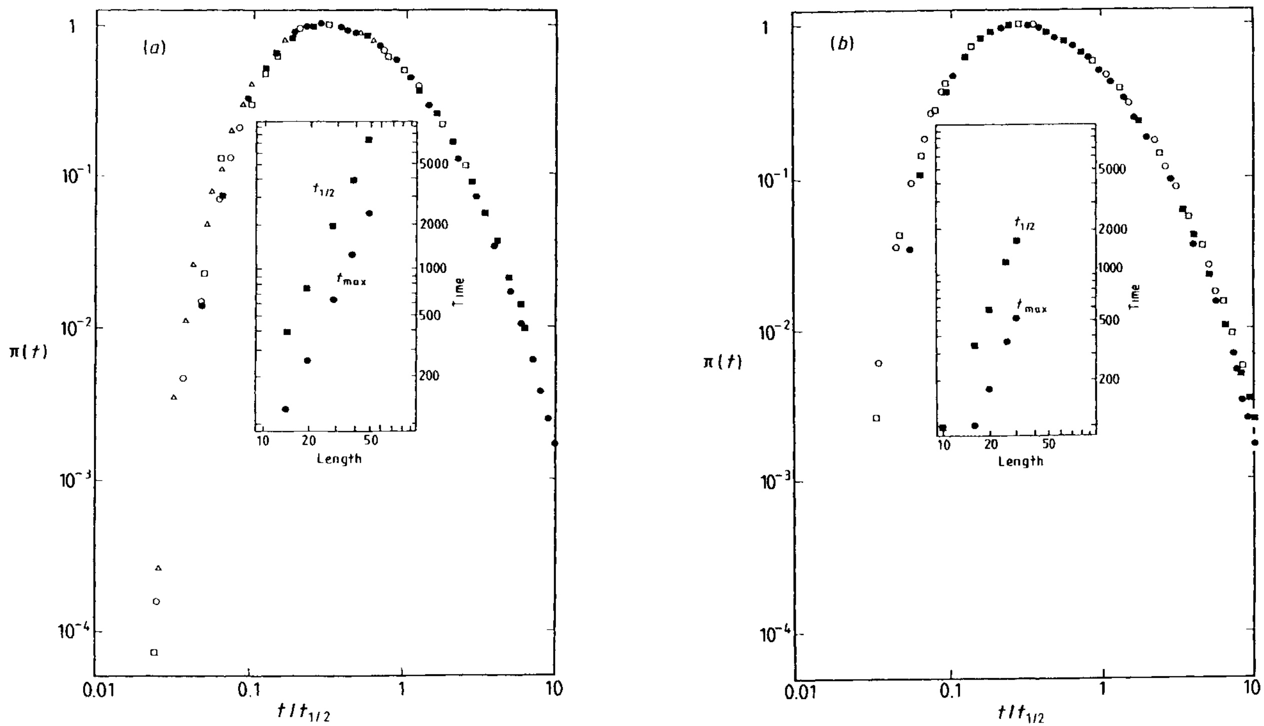

Figure 1. Scaled histogram $\pi(t) = P(t)/P(t_{max})$ of arrival times against scaled time $t/t_{1/2}$ for various chemical distances l. The insert shows the variation of characteristic times with chemical length l. (*a*) Refers to the triangular lattice: $l = 15$ (●), 20 (■), 30 (□), 40 (○) and 50 (△); (*b*) refers to the Cayley tree: $l = 16$ (●), 20 (■), 26 (□), 30 (○).

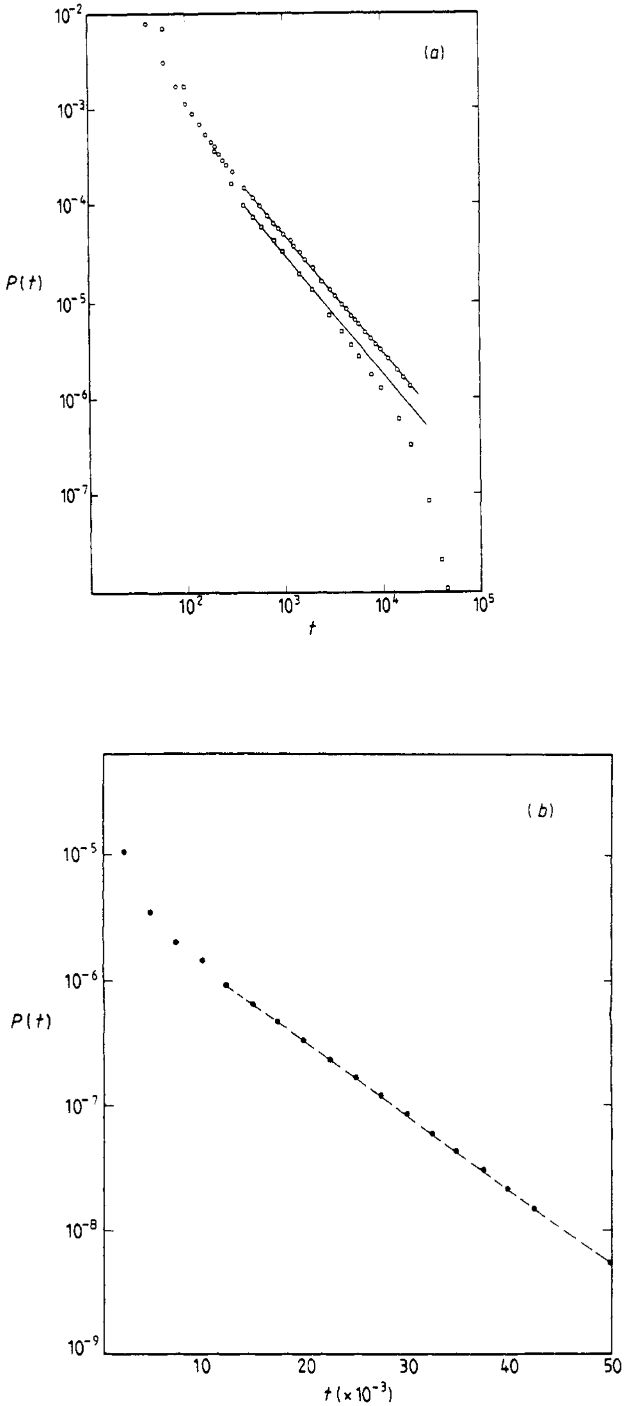

Figure 2. (*a*) Histogram $P(t)$ for the triangular lattice for $l = 35$, $E = 0.8$(○), and for $l = 10$, $E = 0.8$(□). In both cases a power-law regime of $P(t) \sim t^{-1.2}$ is seen. In the case $l = 10$ the exponential decay for $t > 10^4$ is seen clearly in (*b*) where $\ln P(t)$ is plotted against t.

the Bethe lattice. In both cases the data for different l fall into the same curve except for very small $\pi(t)$. Roughly, this curve is a parabola, corresponding to a log-normal distribution of arrival times:

$$\log P(t) \propto [\log(t_{max}) - \log(t)]^2. \tag{1}$$

However, a slight asymmetry is visible, and the log-normal distribution should not be expected to be asymptotically exact. For example, if $t \to \infty$ at fixed l we expect [11] $P(t)$ to decay exponentially, as confirmed by data on $l = 10$ (Cayley tree) for $\pi(t) < 10^{-6}$ (not shown). The first-passage-time distribution $P(t)$ can be related to the distribution of voltage drops between the site at the origin of the walker and a site at chemical distance l. Since for the voltage-drop problem an infinite hierarchy of exponents are needed to characterise the different moments, it is expected that for this case an analogous hierarchy of exponents will characterise the moments $\langle t^n \rangle$.

With a non-zero bias E the results become more complicated. The most probable time t_{max} of arrival shifts, for strong fields ($E \to 1$), towards l, which is the minimum time to traverse l bonds. For t somewhat larger than t_{max}, the arrival probability $P(t)$ falls rapidly. If l is large enough (e.g., $l = 35$ but not $l = 10$) we then see a regime where $P(t)$ decays less strongly, roughly like $1/t$. Finally, for $t \to \infty$ exponential decay is expected, and is seen explicitly in our longest computer run. Figure 2 summarises some of our data. The intermediate regime with its power-law behaviour can be explained as follows. It has been shown [12] that for a walker having a waiting time distribution $\phi(t) \sim t^{-\alpha}$ in a finite system surrounded with traps, the first-passage-time probability $P(t)$ also scales as $t^{-\alpha}$. This is analogous to our case. To calculate α we make use of a recent result [13] found for topological biased diffusion on percolation:

$$P_0(w) \sim \frac{1}{w(\ln w)^{1+\gamma}}. \tag{2}$$

Here $P_0(w)$ is the distribution of transition rates w to pass a dangling end along the backbone of the cluster due to the delays made by visiting in the dangling ends. From (2), and since $w \sim t^{-1}$, we find

$$\phi(t) \sim \frac{1}{t(\ln t)^{1+\gamma}}. \tag{3}$$

This result predicts $P(t)$ to be proportional to $1/t$ with logarithmic corrections. Indeed, the power calculated from figure 2 is $P(t) \sim t^{-1.2}$ which may indicate the effect of logarithmic corrections. The crossover to exponential decay for $t \to \infty$ is also understood: since the system is finite there is a minimum cutoff for equation (2), w_{min}, and, for $t \gg w_{min}^{-1}$, $P(t)$ should decay exponentially. The power-law regime might correspond to $1/f$ noise if Fourier transforms of the current fluctuations are observed [10, 11]. It would be interesting to search for similar effects in other types of bias [14, 15].

This work was supported by the US-Israel Binational Science Foundation. We thank Y Glaser for computational help.

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